

Quasiparticle Transport in the Vortex State of Unconventional Superconductors

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I consider the problem of the vortex contribution to quasiparticle transport in unconventional superconductors with line nodes, and argue that the magnetic field dependence of transport coefficients is fixed by the same scattering processes which limit low-temperature transport. I give no detailed calculations, but show that qualitatively correct results may be obtained in the limit of low temperatures and fields by simple physical arguments, based on estimates of the density of states and relaxation time in analogy with the zero-field case. I conclude with a brief discussion of the problem of anisotropy of the field dependence and influence of experimental geometry.

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Introduction. For some time, transport measurements in the Abrikosov vortex state of cuprate superconductors have exhibited magnetic field and temperature dependences which seemed difficult to incorporate into the standard picture developed for classic superconductors. [1] In this model, if vortices are pinned, quasiparticles carry all electrical current and entropy, and their transport is limited exclusively by scattering from vortex cores. The cores play a crucial role because of a relatively high density of bound states created there by the local order parameter suppression, and because extended quasiparticle states are completely gapped.

In the past three or four years, a consensus has been established that the order parameter in the hole-doped cuprate superconductors has *d*-wave symmetry. [2] It is therefore natural to ask if the new symmetry class, and in particular the existence of gap nodes on the Fermi surface, can invalidate the “vortex scattering” model used to analyze quasiparticle transport in classic superconductors. In this paper, a summary of two lectures given at the February 1998 Workshop on Strongly Correlated Systems of the Asia Pacific Center for Theoretical Physics, I argue that this is in fact the case: that the existence of low-energy quasiparticle states due to the order parameter nodes allows one to neglect vortex scattering at small temperatures and fields, and focus instead on the rearrangement of nodal quasiparticle occupation numbers occasioned by the Doppler shifts of quasiparticle energies in the superflow field of the vortex lattice. Relevant scattering processes are the same which dominate zero-field transport, leading to the possibility that the successful “dirty *d*-wave” theory of the $H = 0$ superconducting state transport in HTSC can be extended to the vortex state without the introduction of further parameters. Predictions of the theory for the field dependence of, e.g., the thermal conductivity in the vortex state, are consistent with measurements made to date at higher temperatures. At low temperatures, where the predictions are drastically different from those of the vortex scattering model, preliminary experimental results support the theory, but more experiments are needed.

I begin by providing the qualitative background necessary to understand the temperature dependence of $H = 0$

transport coefficients in the cuprates. I then summarize the arguments why extended quasiparticle states may be assumed to dominate the vortex bound states, and discuss the semiclassical approximations used to treat them. I show that at $T = 0$, transport coefficients *increase* quasilinearly with field, but at finite temperatures exhibit a nonmonotonic field dependence. For the most part I restrict myself in this paper to qualitative arguments which can be shown to provide the same results as obtained by a more complete treatment; in doing so I am mostly summarizing work with C. Kübert which has appeared elsewhere. [3–5] However, in section 4, I consider the influence of relative configuration of magnetic field and current, and give as a concrete example new results for thermal transport as a function of field in the basal plane of a $d_{x^2-y^2}$ superconductor.

I. INTRODUCTION: ZERO-FIELD TRANSPORT

At sufficiently low temperatures, the quasiparticle transport time will be determined by impurity scattering. If one restricts one’s attention to the *s*-wave scattering channel, but assumes an unconventional superconducting state in which $\sum_k \Delta_k = 0$, Pethick and Pines [6] showed in the context of heavy fermion superconductivity that $1/\tau(\omega)$ varies as $N(\omega)$ in the conventional weak scattering (Born) limit, and as $N(\omega)^{-1}$ in the strong scattering (unitarity) limit. The strength of individual scattering events therefore has profound consequences for transport coefficients L , which may be estimated as

$$L \sim \int \left(\frac{-\partial f}{\partial \omega} \right) N(\omega) \tau(\omega) \sim N(T) \tau(T) \quad (1)$$

$$\sim \begin{cases} N(T)^2 & \text{unitary} \\ L_N \simeq \text{const.} & \text{Born} \end{cases}$$

where $L_N = L(T_c)$.

Experimentally, in both heavy fermion and cuprate superconductors, it is found that in good samples, bulk transport coefficients are orders of magnitude smaller than their normal state counterparts, thus leading to the (hitherto microscopically unjustified) *ansatz* of unitarity

scattering by simple defects in these systems. Formally, such calculations are carried out either in the kinetic equation approach of Pethick and co-workers, [7] or in the self-consistent t -matrix approximation (SCTM). [8,9] In the unitarity limit, both approaches yield similar results for temperatures and energies above a scale $\gamma \sim \sqrt{\Delta_0 \Gamma}$, where Δ_0 is the gap maximum and Γ is the impurity scattering rate in the normal state. At low temperatures, only the SCTM yields true gapless behavior (finite residual density of states $N(0)$), as well as the remarkable prediction of universal transport, [10], recently confirmed by experiment in thermal conductivity measurements on Zn -doped YBCO. [11] Briefly, this occurs because zero-energy quasiparticle states acquire a residual broadening $1/\tau(0) \equiv \gamma$ in the presence of impurities due to the nodes of the gap, and the residual density of states is also found to scale with γ/Δ_0 . Thus L is generically of order $N_0(\gamma/\Delta_0)/\gamma \sim N_0/\Delta_0$, i.e. L is independent of the impurity concentration to leading order in the disorder Γ . To give an explicit example, in a $d_{x^2-y^2}$ state, $\Delta_k = \Delta_0 \cos 2\phi$ over a model cylindrical Fermi surface, the thermal conductivity in zero applied magnetic field for heat current \mathbf{j}_Q in the plane is approximately

$$\frac{\kappa^{el}(T)}{\kappa_{00}} \simeq 1 + \frac{7\pi T^2}{5\Delta_0 \Gamma} \left(\ln^2 \frac{1.14\Delta_0}{T} + \frac{\pi^2}{4} \right). \quad (2)$$

where $\kappa_{00}/T \equiv \pi N_0 v_F^2 / (6\Delta_0)$ is the universal thermal conductivity. [12–14]

At higher temperatures in the cuprates, the mean free path due to inelastic scattering becomes smaller than that due to impurity scattering. DC transport coefficients generically then show the following structure: a rise at low temperatures from the universal value to a maximum at intermediate temperatures, followed by a decrease to the normal state value at T_c , typically an order of magnitude or so less than the peak in clean systems, but an order of magnitude greater than the coefficients at the lowest temperatures. (See Fig. 1) The existence of the maximum has been qualitatively understood for some time as arising from the competition between a collapsing inelastic scattering rate $1/\tau_{inel}$ below T_c due to an opening of a gap in the spectrum of excitations responsible for inelastic scattering, and the decrease of the number of quasiparticle carriers at low temperatures. [15,16] At low temperatures there may be problems with the details of this simple argument, due to the energy dependence of the scattering rate due to impurities, $1/\tau_{imp}$ (as discussed above), but the overall features are well-described by models which add impurity and inelastic scattering rates incoherently and include the collapse of the inelastic scattering below T_c in some form. My co-workers and I argued in [17] that at sufficiently low temperatures, when quasiparticle states are well-defined, $1/\tau_{inel}$ should vary as T^3 for the simple reason that electron-electron collisions in a normal Fermi liquid with constant particle number leads to T^2 , and there is one additional factor of the temperature arising

from the linear d -wave density of states. This estimate is borne out in a weak-coupling Hubbard model calculation of $1/\tau$, [18] which yielded excellent results in comparison to both low-frequency microwave conductivity [17] and thermal conductivity [19] measurements, including disorder dependence, over the entire temperature range.

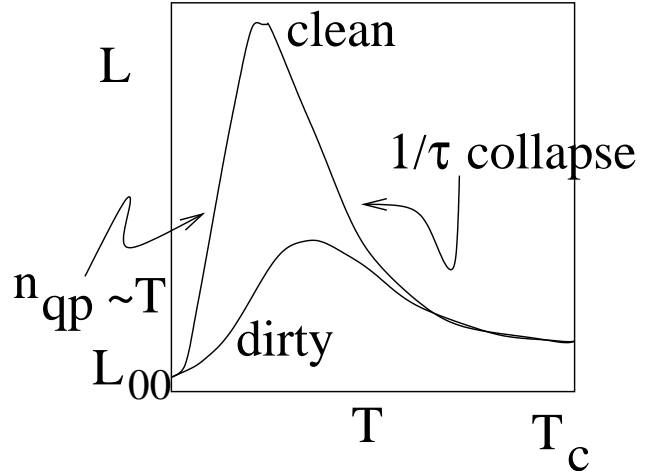


FIG. 1. Schematic behavior of DC transport coefficients in the cuprates.

II. QUASIPARTICLES IN THE UNCONVENTIONAL VORTEX STATE

I now turn to the question of transport in the vortex state, focussing for consistency on longitudinal thermal conductivity measurements, although microwave conductivity measurements should share many of the same features. Thermal conductivity as a probe of the quasiparticle system has the advantage that in most cases of current interest vortex motion may be ignored. While the phonon contribution is often very difficult to determine, it seems likely that it is field independent, so the quasiparticle system is directly probed by measurement of the field dependent part $\delta\kappa(T; H)$. This question is still open, however. There are several measurements of κ at temperatures down to 10K or 20K in YBCO-123 and BSCCO-2212, a typical recent one being that of Freimuth, [20] which shows at fixed temperature $T \sim 0.1T_c$ that the thermal conductivity decreases with application of a field with sublinear power or log behavior. The qualitative behavior at higher temperatures is not inconsistent with the conventional vortex scattering model, in the sense that the quasiparticle mean free path shortens with increasing field as might be expected if the vortices are moving closer together. However, the form of the field dependence is unusual, since, within the picture just alluded to, the number of scatterers should scale linearly

with H . Furthermore, recent measurements at lower T on both YBCO [21] and UPt_3 , [22] a heavy fermion system also with an order parameter with line nodes, have exhibited a thermal conductivity *increasing* quasilinearly with field. This can never be explained within a vortex scattering picture.

Vortex scattering in the cuprates is in fact likely to be negligible at low fields for several reasons. The density of bound states in a state with gap nodes was found to be significantly smaller than the density of extended states by Volovik [23] when averaged over the entire vortex. This calculation accounted for the gap nodes, but treated the bound states as a continuum and thus must be regarded as an upper bound to the true contribution. In fact, in the cuprates the bound state separation or “minigap” $\delta \sim \Delta_0 k_F^{-1} / \xi_0$ is very large compared to classic superconductors, and it is unlikely at low T that more than a single state/vortex is occupied. The net cross section for quasiparticle scattering is therefore likely to be very much smaller than in the classic case. Finally, I note that at low temperatures the picture of independent scattering events by vortices must break down in the perfect vortex lattice. The quasiparticle-vortex mean free path ℓ_{vortex} will then arise solely from the disorder-induced or residual thermally excited fluctuations of the ideal periodic Abrikosov lattice. At $H = 1T$, the intervortex distance of $2R = \xi_0(2\pi)^{1/2}a^{-1}(H_{c2}/H)^{1/2}$, which provides an absolute lower bound for ℓ_{vortex} , is about 25 times the coherence length ξ_0 , or perhaps 400\AA in HTSC. While in YBCO the mean free path due to defect scattering and electron-electron collisions ℓ_{tr} at low T may be as long as 3000\AA , [16], in BSCCO ℓ_{tr} is a factor of ten or so smaller, [24] and therefore probably smaller than ℓ_{vortex} over most of the field range of $10T$ or so in current experiments. Note that this same argument applies to Andreev scattering by the vortex flow field as considered in Ref. [25], since again vortex disorder, rather than the intervortex distance, sets the mean free path.

If mean free paths due to vortex scattering are indeed much longer than in classic superconductors, other scattering processes, such as impurities and electron-electron collisions, will limit the mean free path at low fields $H \ll H_{c2}$. I will simply assume this for the discussion below, and beg the question of the range of fields for which such an assumption may be justified.

In his seminal paper, Volovik showed that the dominant contribution of the extended quasiparticle states to the density of states took the form

$$N(0; H) \sim N_0 \sqrt{\frac{H}{H_{c2}}}. \quad (3)$$

Such a term was indeed extracted from specific heat data on YBCO by Moler et al., [26] at the time a crucial piece of evidence in support of the identification of d -wave symmetry in YBCO. This result can be reproduced easily if it is recalled that 1) the density of states in the pure d -wave

state varies as $N(\omega) \sim \omega/\Delta_0$ at low energies $\omega \ll \Delta_0$; and 2) that a quasiparticle of momentum \mathbf{k} moving in the superflow field of a single vortex, $\mathbf{v}_s(\mathbf{r}) \simeq \hbar/(2mr)\hat{\alpha}$ (α is the azimuthal angle in real space, e.g. in the plane if $H \parallel \hat{c}$) experiences a Doppler shift $\omega \rightarrow \omega - \mathbf{v}_s \cdot \mathbf{k}$ in the lab frame. We then have at $\omega = 0$

$$N(0; H) \simeq \langle \frac{|\mathbf{v}_s \cdot \mathbf{k}|}{\Delta_0} \rangle_H \sim \frac{1}{R^2} \int_0^R dr r \left(\frac{\xi_0}{r} \right) \quad (4)$$

where in the second approximate equality an average $\langle \dots \rangle_H$ has been performed over the vortex unit cell, of size R . Since the integrand is constant, $N(0; H) \sim R^{-1} \sim \sqrt{H}$ follows immediately.

III. THERMAL CONDUCTIVITY AT $H > 0$

Similar arguments can be applied [5] to the kernel of the thermal conductivity Kubo formula. At $T = 0$, analogous to the discussion in Sec. 1, we have

$$N(0; \mathbf{v}_s) \sim v_s \quad ; \quad \tau(\mathbf{v}_s) \sim N(0; \mathbf{v}_s) \sim v_s \quad (5) \\ \Rightarrow \frac{\kappa}{T} \sim \langle v_s^2 \rangle_H \sim H \log H,$$

since the integrand now varies as $1/r$ rather than as a constant. Note in this case the average requires a lower cutoff, of order ξ_0 , which reflects our ignorance of how the extended quasiparticle wavefunctions vanish in the core region. Furthermore, we implicitly assumed in Eq. (5) that the local conductivity $\kappa(\mathbf{r})$ depended only on r : this is true only for the special case $\mathbf{j}_Q \parallel \mathbf{H}$. A treatment of the opposite case, $\mathbf{j}_Q \perp \mathbf{H}$ shows that the measured (averaged) κ is much smaller since one necessarily adds thermal resistances in series rather than in parallel in this case. [5] A full numerical evaluation shows a behavior slightly more linear in field than given by Eq. (5), due to higher contributions of order v_s^4 , which give rise to true linear terms when averaged.

At higher temperatures, terms decreasing in field arise due to 1) logarithmic energy dependence of the unitarity limit scattering rate (neglected in Eq. (1)), and 2) inelastic scattering. The competition between increasing and decreasing contributions leads in this case to a minimum in the field dependence, at $H \sim T^2$ in the case of 1). Determination of the exact form including inelastic scattering has not yet been worked out in detail, but we know these effects are required for the understanding of cuprates in zero field above about 1K or so. Simple estimates show that the minimum in this more realistic case is shifted out to fields higher than heretofore measured; one can thus via this method understand both the magnitude and form of the (apparently) monotonically decreasing field dependences reported in, e.g. Ref. [20].

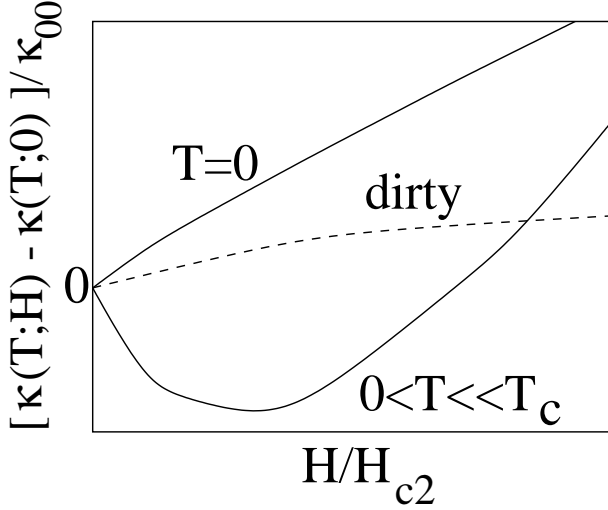


FIG. 2. Schematic behavior of $\kappa(T;H)/T$ at various temperatures for a system with order parameter line nodes. Solid lines: clean system at $T = 0$, $0 < T < T_c$; dashed line: dirty system at $T = 0$.

IV. ANISOTROPY

In the above estimates, I have suppressed consideration of spatial angular degrees of freedom in order to extract the qualitative field dependence. In general, the four-fold structure of the gap Δ_k in momentum space fixed to the crystal field axes will induce an angular anisotropy in the current response due to the coupling $\mathbf{v}_s \cdot \mathbf{k}$. On the other hand, I have already mentioned in Sec. III a further source of anisotropy due to the different types of spatial averaging necessary for different directions of \mathbf{j}_Q relative to \mathbf{H} . By itself, this leads generically to a maximum in the thermal conductivity when $\mathbf{j}_Q \parallel \mathbf{H}$, consistent with observations by Yu et al. [25] and Aubin et al. [27]. Unfortunately, this qualitative feature is also common to vortex scattering theories. It would be useful to consider an effect which is clearly due entirely to the gap anisotropy.

Such an example is provided by taking the thermal current \mathbf{j}_Q parallel to the field \mathbf{H} in the plane, with both making an angle ϵ with respect to the x -axis. In a coordinate system in which the quasiparticle momentum is $\mathbf{k} = (\cos \phi, \sin \phi, 0)$, the superfluid velocity is then $\mathbf{v}_s = (-\sin \epsilon \sin \alpha, \cos \epsilon \sin \alpha, \cos \alpha)$ (reminder: α is the vortex winding angle). The quasiparticle energy shift is then

$$\begin{aligned} \mathbf{v}_s \cdot \mathbf{k} &= v_s k_F \sin \alpha \sin(\phi - \epsilon) \\ &\simeq \frac{v_s k_F}{\sqrt{2}} \sin \alpha (\pm \cos \epsilon \pm \sin \epsilon), \end{aligned} \quad (6)$$

where the final approximate equality is justified because the kernel in the Kubo formula is strongly peaked at the nodal momenta \mathbf{k}_n with $\phi = \pi/4, 3\pi/4, \dots$. Repeating the previous calculation, including the

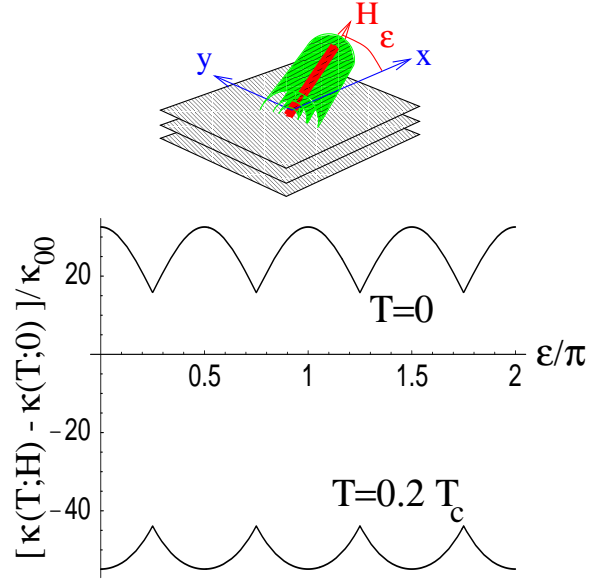


FIG. 3. Anisotropy of $[\kappa(T;H) - \kappa(T;0)]/\kappa_{00}$ as function of field direction ϵ (see insert) relative to the crystal axes for $T = 0$ and $T = 0.2T_c$ and $H/H_{c2} = 0.01$. Weak coupling value $\Delta_0/T_c = 2.14$ was assumed for the $d_{x^2-y^2}$ order parameter.

angular average, and remembering to symmetrize over all nodes, one finds (up to factors of order unity)

$$\begin{aligned} \frac{\kappa(H)}{T} &\sim \left\langle \left(\frac{\mathbf{v}_s \cdot \mathbf{k}}{\Delta_0} \right)^2 \right\rangle_H \\ &\sim \frac{1}{R^2} \int_{\xi_0}^R dr r \left(\frac{\xi_0}{r} \right)^2 \int_0^{2\pi} d\alpha \sin^2 \alpha \max(\cos^2 \epsilon, \sin^2 \epsilon) \end{aligned} \quad (7)$$

so the thermal conductivity is seen at $T = 0$ to exhibit 4-fold oscillations in the field dependence as a function of field angle in the plane with a *minimum* in the nodal directions. In Figure 3, I plot a full evaluation of the low-energy approximation given in Ref. [5] as a function of angle in the basal plane. Note that a traditional vortex scattering model would predict no effect in this case. At a higher temperature, the model of Ref. [5] predicts a decreasing $\kappa(H)$ for the field specified, and consequently *maxima* along the nodal directions; thus the oscillations appear similar to those observed in an experiment with slightly different geometry (but with \mathbf{H} in the plane) by Yu et al. [25], and attributed to Andreev scattering. A comparison of these two effects is in progress.

V. CONCLUSIONS

I have tried to give a somewhat pedagogical summary of the ideas currently used to interpret transport experiments at low temperatures in d -wave and other unconventional superconducting states. Back-of-the-envelope

estimates for the field dependence of thermal conductivity in a field were shown to reproduce the more detailed calculations of recent work suggesting that the field dependence of transport coefficients in heavy fermion and cuprate superconductors can be well described by ignoring vortex scattering and including only $H = 0$ scattering processes, but shifting quasiparticle energies in the vortex superflow field. Fourfold oscillations of transport coefficients as a function of the angle between magnetic field direction $\mathbf{H} \parallel \mathbf{j}$ and the crystal axes in the plane of a $d_{x^2-y^2}$ superconductor were shown to be a natural consequence of this picture.

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